Standard: Use matrices to represent and manipulate data, multiply matrices by scalars, add, and subtract.

|  |  |
| --- | --- |
| $\begin{matrix} Master check quiz&1\end{matrix}\begin{matrix} 2 &3 &4\end{matrix}$  $\begin{matrix}Granger, H\\Malfoy, D\\Potter, H\end{matrix}$ $\left[\begin{matrix}10&10&10\\7&8&0\\8&7&7\end{matrix} \begin{matrix}10\\0\\9\end{matrix}\right]$ | Matrices are used to represent and manipulate data. A gradebook, sports statistics, or spreadsheets are examples of organizing data as a matrix. Matrices are referred to by their sizes. The size of a matrix is given in the form of a dimension, like a piece of wood is 2X4. |

The matrix dimensions are X . Row: Column:

The individual numbers inside a matrix are called entries and can be labeled as follows:

$\left[\begin{matrix}a\_{11}&a\_{12}&a\_{13}\\a\_{21}&a\_{22}&a\_{23}\end{matrix}\right]$

Adding and subtracting matrices:

Example:

|  |  |
| --- | --- |
| $$\left[\begin{matrix}2&6&4\\9&n&8\end{matrix}\right]+\left[\begin{matrix}5&7&1\\9&-n&3n\end{matrix}\right]=$$ | $$\left[\begin{matrix}4&7&4\\5n&n&8\end{matrix}\right]-\left[\begin{matrix}5&12&1\\9&-n&3\end{matrix}\right]=$$ |

Scalar Multiplication:

Example:

|  |  |
| --- | --- |
| Find -1AA$=\left[\begin{matrix}1&2\\3&4\end{matrix}\right]$ | Find $\frac{2}{a}$B$$\left[\begin{matrix}3&-2\\4&7\end{matrix}\right]$$ |
| Find 4(A+B)A$=\left[\begin{matrix}3&4\\1&6\end{matrix}\right]$ B$=\left[\begin{matrix}2&-1\\5&-6\end{matrix}\right]$ | Find A-3BA$=\left[\begin{matrix}-2&4\\3&1\end{matrix}\right]$ B$=\left[\begin{matrix}1&2&3\\4&5&6\end{matrix}\right]$ |

Standard: Solve systems of linear equations using matrices.

Matrices can be used to represent a system of linear equations, and then used to solve the system. The system is solved when the matrix is in reduced row echelon form

$\left[\begin{matrix}1&0&\#\\0&1&\#\end{matrix}\right]$

1.
2.
3.

There are three types of elementary row operations:

1.
2.
3.

Example:

|  |  |
| --- | --- |
| $$3x+y=1$$$$-x+y=3$$ | $$2y-6x=2$$$$y=x-3$$ |

Standard: Use technology to solve systems using matrices.

1. 2nd MATRX
2. Go to EDIT
3. Press ENTER
4. Now type in the dimension of the matrix, and type in the values of the matrix and pres ENTER after you input each value.
5. Press 2nd QUIT
6. Pres 2nd MATRX
7. Go to MATH and scroll down to B:rref(
8. Press ENTER
9. Press 2nd MATRX
10. Ensure the cursor is on 1:[A] and press ENTER
11. Add an ending parenthesis
12. Press ENTER

Example:

$$x+2y+z=4$$

$$3x+8y+7z=20$$

$2x+7y+9z=23$

Standard: Multiply matrices of appropriate dimension

To find the product of two matrices:

1.
2.

|  |  |
| --- | --- |
| Find ABA$=\left[\begin{matrix}1&5\\3&7\end{matrix}\right]$ B=$\left[\begin{matrix}4&1&-2\\3&-1&1\end{matrix}\right]$ | Find BAA$=\left[\begin{matrix}1&5\\3&7\end{matrix}\right]$ B=$\left[\begin{matrix}4&1&-2\\3&-1&1\end{matrix}\right]$ |

Standard: Understand that matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

Properties of multiplication for square matrices:

Commutative:

Distributive:

Associative:

A$=\left[\begin{matrix}2&-1\\3&2\end{matrix}\right]$ B$=\left[\begin{matrix}-4&2\\1&3\end{matrix}\right]$ C$=\left[\begin{matrix}0&1\\2&3\end{matrix}\right]$

 Distributive Associative

$B+C=\left[\begin{matrix}-4&3\\3&6\end{matrix}\right] A\left(B+C\right)=\left[\begin{matrix}-11&0\\-6&21\end{matrix}\right]$ $BC=\left[\begin{matrix}4&2\\6&10\end{matrix}\right]$ $A\left(BC\right)=\left[\begin{matrix}2&-6\\24&26\end{matrix}\right]$

$AB=\left[\begin{matrix}-9&1\\-10&12\end{matrix}\right] AC=\left[\begin{matrix}-2&-1\\4&9\end{matrix}\right]$ $AB=\left[\begin{matrix}-9&1\\-10&12\end{matrix}\right] \left(AB\right)C=\left[\begin{matrix}2&-6\\24&26\end{matrix}\right]$

$$AB+AC=\left[\begin{matrix}-11&0\\-6&21\end{matrix}\right]$$

Standard: Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

Zero and Identity Matrices: When we work with real numbers we notice some patterns when multiplying by 0 and multiplying by 1. We see similar patterns when we multiply matrices by the zero and the identity matrices.

Identity matrix:

$$I=\left[\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right]$$

Zero matrix:

|  |  |
| --- | --- |
| Find the product of $IA$$$A=\left[\begin{matrix}5&8\\4&2\end{matrix}\right]$$ | Multiply A by the zero matrix |

Properties of zero and the identity matrix: For real numbers the following two rules are familiar: If ab=ac and $a\ne 0$, then b=c, and if ad=0 then either a=0 or d=0.

$A=\left[\begin{matrix}4&1&-2\\3&1&-1\end{matrix} \begin{matrix}7\\ 5\end{matrix}\right]$ $B=\left[\begin{matrix}1&5\\3&-1\\-2&4\\2&-3\end{matrix}\right]$ $C=\left[\begin{matrix}3&4\\2&1\\-2&3\\1&-3\end{matrix}\right]$ $D=\left[\begin{matrix}-2&1\\1&-2\\0&1\\1&0\end{matrix}\right]$

$$AB=\left[\begin{matrix}25&-10\\18&-5\end{matrix}\right]$$

$$AC=\left[\begin{matrix}25&-10\\18&-5\end{matrix}\right]$$

$$AD=\left[\begin{matrix}0&0\\0&0\end{matrix}\right]$$

Determinant: The purpose of a determinant is to capture in one number the essential features of a matrix.

Recall: A **square matrix** has the same number of rows and columns.

**Finding the Determinant of a 2 x 2 Matrix**

The determinant of the matrix is



“*Notice that the matrix is now surrounded by straight bars instead of brackets. The straight bars indicate determinant. So when dealing with matrices and you come across straight bars around the matrix, DO NOT think absolute value. Think DETERMINANT.”*

**Example 1:** Find the following:

 a. $\left|\begin{matrix}2&-2\\3&6\end{matrix}\right|=$

 b. $\left|\begin{matrix}-4&-2\\6&3\end{matrix}\right|=$

 *Can you identify a relationship between the rows/columns of this matrix?*

 c. $\left|\begin{matrix}1&5\\6&-4\end{matrix}\right|=$

Finding the determinant on a calculator

1. Press 2nd MATRX
2. Go to EDIT and enter the dimensions and entries from the matrix
3. Press 2nd QUIT
4. Press 2nd MATRX
5. Go to MATH and make sure the cursor is on 1:det( then press enter
6. Press 2nd MATRX and make sure the cursor is on 1:[A] and press enter

Inverse of a Matrix: In real numbers the inverse of a real number *a* was the number $a^{-1}$, and when you multiply $a$ and $a^{-1}$ equal 1. $a^{-1} $is the reciprocal of a. The inverse of a square matrix when multiplied by the matrix will produce the identity matrix.

Finding the inverse:

Requirements

1.

Finding the Inverse of a 2x2 Matrix:

1.
2.
3.

Example:

$$\left[\begin{matrix}7&-2\\3&5\end{matrix}\right]$$

Finding the Inverse of a larger matrix:

1.

Example:

$$\left[\begin{matrix}0&1&-3\\-2&6&-1\\5&-4&2\end{matrix}\right]$$