

Name _____ Due Date _____ Period _____

11.2 End behavior (F.IF.7)

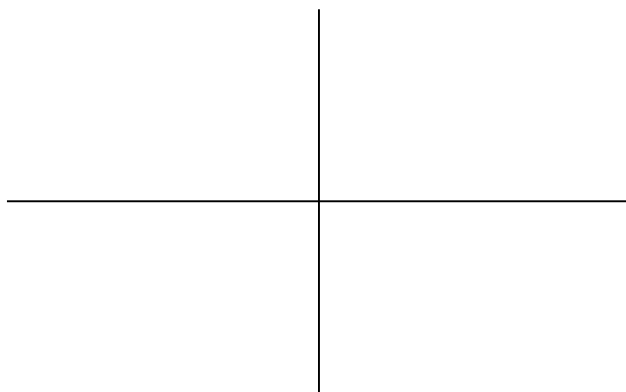
A good window for all the graphs will be $[-10, 10] \times [-25, 25]$

In this lab, we are looking at the end behavior of polynomial graphs, i.e. what is happening to the y values at the (left and right) ends of the graph.

In other words, we are interested in what is happening to the y values as we get really large x values and as we get really small x values.

To practice recording end behavior, let's look at $y = x$

Sketch a graph of it. (It would be nice to be able to do from memory, but use your calculator if you need.)



Notice how the y values soar off toward infinity on the right end (as x values get really big) and the y values soar off toward negative infinity on the left end (as x values get really small).

We show this by writing "as $x \rightarrow -\infty, y \rightarrow -\infty$ and as $x \rightarrow \infty, y \rightarrow \infty$ ". This is read "as x approaches negative infinity, y approaches negative infinity and as x approaches positive infinity, y approaches positive infinity". Notice that the first part of this talks about the left end of the graph and the second part of this talks about the right end of the graph. Sign your initials next to this to show that you read this part.

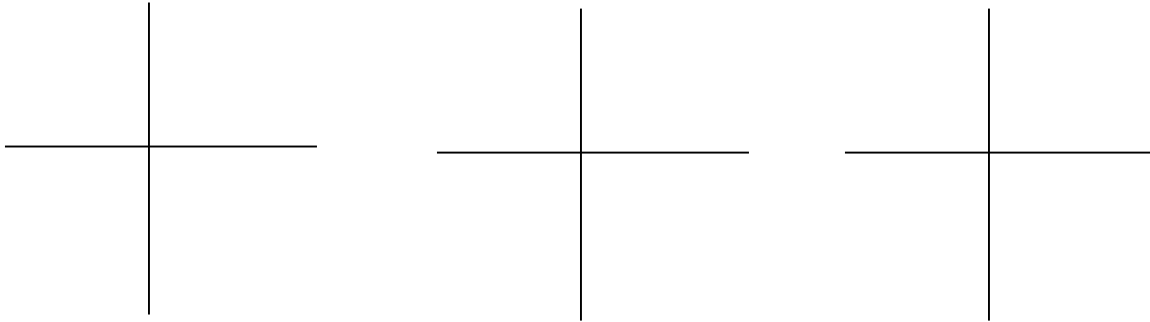
End Behavior:

$$\text{As } x \rightarrow -\infty, y \rightarrow \underline{-\infty}$$

$$\text{As } x \rightarrow \infty, y \rightarrow \underline{\infty}$$

This worksheet will guide you through looking at the end behaviors of several polynomial functions. At the end, we will generalize about all polynomial functions. Try to mimic the general shape and the end behavior of the graphs.

1. Graph $y = 2x^3 + 4x$, $y = 3x^5 - 6$, and $y = x^7 + 2x^3 - 5x^2 + 2$ on the three planes below.



What is the end behavior of all three graphs above? Use the notation demonstrated above.

End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

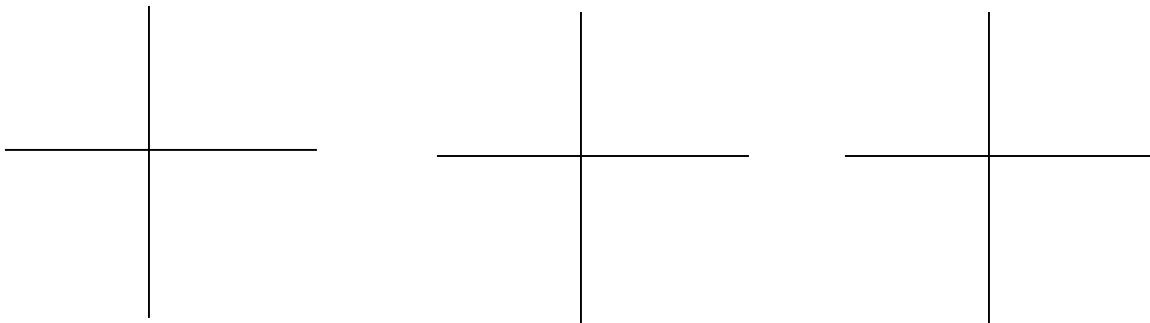
As $x \rightarrow \infty$, $y \rightarrow$ _____

End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

2. Graph $y = -3x^5 - 5x^2 + 3x - 4$, $y = -4x^3 + 2x^2 + 3$, and $y = -x^7 + 2x^2 + 3x - 4$ on the three planes below.



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

End Behavior:

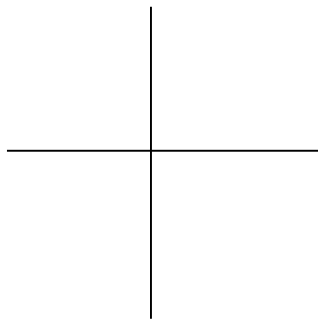
As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

3. **The two things that determine the end behavior of a polynomial are the degree (whether it's even or odd) and the leading coefficient (whether it's positive or negative). Look over your work for questions one and two to verify this.** Make a generalization based on your work so far...

	Leading coefficient is negative	Leading coefficient is positive
Degree is odd		

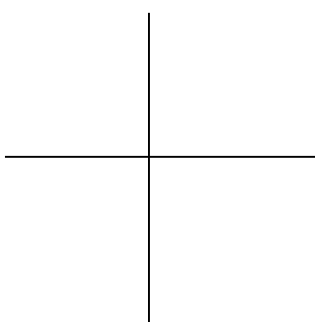
4. Graph $y = 2x^4 + 4x$, $y = 3x^6 - 6$, and $y = x^2 + 2x + 2$ on the three planes below.



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

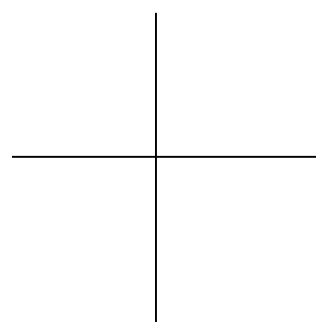
As $x \rightarrow \infty$, $y \rightarrow$ _____



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

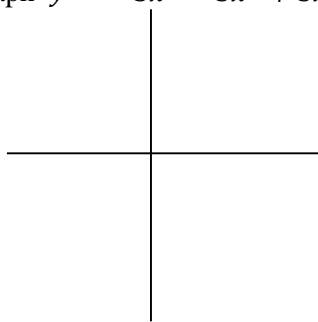


End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

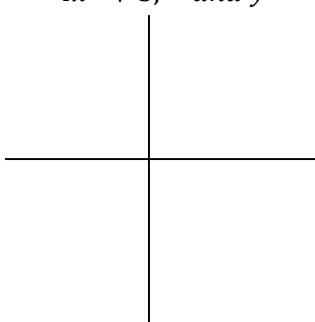
5. Graph $y = -3x^8 - 5x^2 + 3x - 4$, $y = -4x^2 + 3$, and $y = -x^4 + 2x^2 + 3x - 4$ on the three planes below.



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

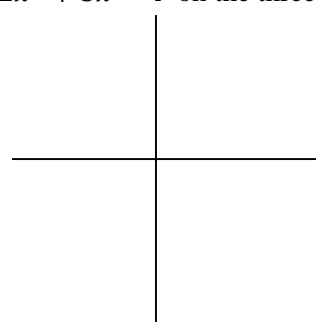
As $x \rightarrow \infty$, $y \rightarrow$ _____



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____



End Behavior:

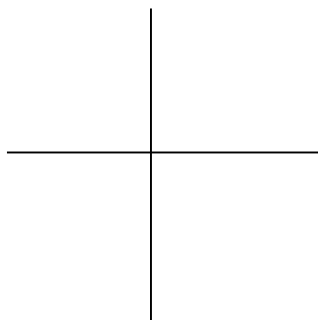
As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

6. The two things that determine the end behavior of a polynomial are the degree (whether it's even or odd) and the leading coefficient (whether it's positive or negative). Look over your work for questions one and two to verify this. Make a generalization based on your work so far...

	Leading coefficient is negative	Leading coefficient is positive
Degree is even		

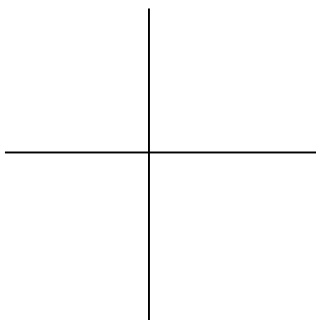
7. Graph $y = 3(3)^x$, $y = 6(2)^x$, and $y = 4^x$ on the three planes below.



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

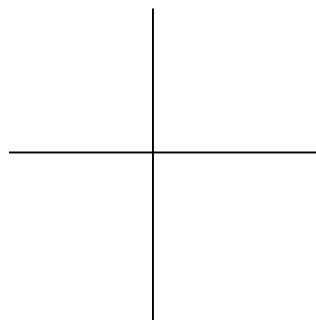
As $x \rightarrow \infty$, $y \rightarrow$ _____



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

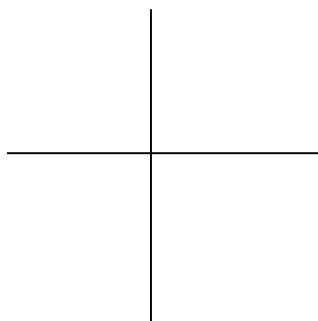


End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

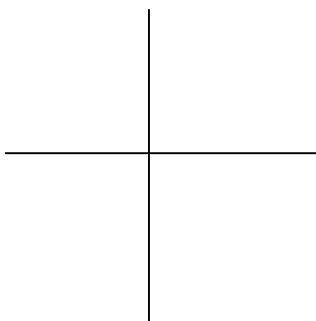
8. Graph $y = -3(3)^x$, $y = -6(2)^x$, and $y = -1(4)^x$ on the three planes below.



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

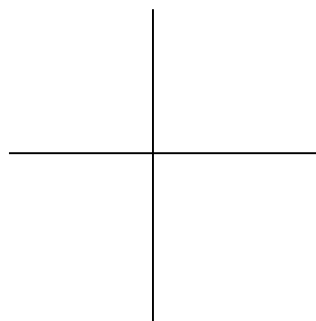
As $x \rightarrow \infty$, $y \rightarrow$ _____



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____



End Behavior:

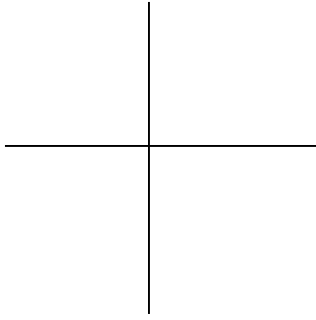
As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

9. The two things that determine the end behavior of an exponential are the rate (growth or decay) and the initial value (whether it's positive or negative). Look over your work for questions one and two to verify this. Make a generalization based on your work so far...

	Initial value is negative	Initial value is positive
Rate is growth		

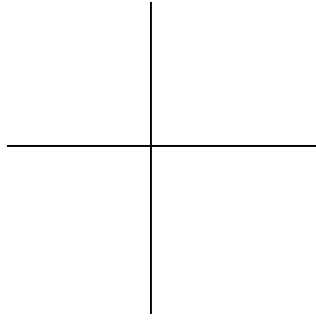
10. Graph $y = 3\left(\frac{2}{3}\right)^x$, $y = 6\left(\frac{1}{2}\right)^x$, and $y = \left(\frac{3}{4}\right)^x$ on the three planes below.



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

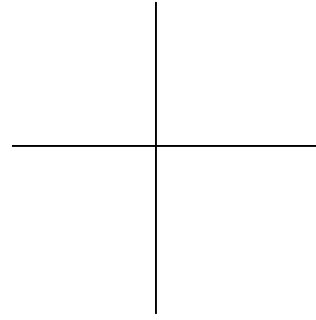
As $x \rightarrow \infty$, $y \rightarrow$ _____



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

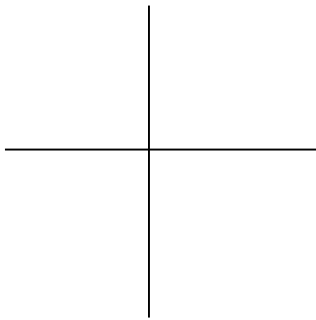


End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

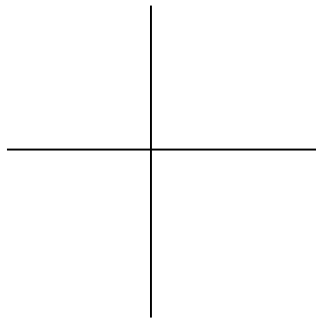
11. Graph $y = -3\left(\frac{2}{3}\right)^x$, $y = -6\left(\frac{1}{2}\right)^x$, and $y = -1\left(\frac{3}{4}\right)^x$ on the three planes below.



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

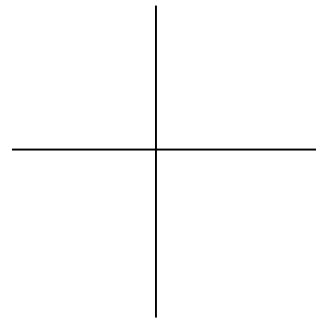
As $x \rightarrow \infty$, $y \rightarrow$ _____



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____



End Behavior:

As $x \rightarrow -\infty$, $y \rightarrow$ _____

As $x \rightarrow \infty$, $y \rightarrow$ _____

12. The two things that determine the end behavior of an exponential are the rate (growth or decay) and the initial value (whether it's positive or negative). Look over your work for questions one and two to verify this. Make a generalization based on your work so far...

	Initial value is negative	Initial value is positive
Rate is decay		